

## Multiple spins systems

### AX system

Calculate the coupling constant  $^1J(^{31}\text{P}^{77}\text{Se})$  to one decimal place, the chemical shift in Hz of phosphorus in molecules containing  $^{77}\text{Se}$ , and this shift in ppm, to two decimal places.

- 1(a)  $^1J(^{31}\text{P}^{77}\text{Se}) = 760.6 \text{ Hz}$   
 1(b)  $\nu_{\text{P}}(^{77}\text{Se}) = 4415.6 \text{ Hz}$   
 1(c)  $\nu_{\text{P}}(^{77}\text{Se}) = 36.34 \text{ ppm}$

### AB system

**Question:** The four lines of the  $^{31}\text{P}$  AB spectrum shown above are at

18906.223, 18679.424, 18280.477, 18053.680 Hz.

1. Calculate  $\nu_{\text{A}}$ ,  $\nu_{\text{B}}$  and  $J_{\text{AB}}$  in Hz, all to one decimal place.

$$J_{\text{AB}} = 226.8 \text{ Hz}; \nu_{\text{M}} = 18479.95 \text{ Hz (152.1 ppm)}; \delta_{\text{AB}} = 583.20 \text{ Hz (4.80 ppm)}$$

$$\nu_{\text{A}} = \nu_{\text{M}} - \delta_{\text{AB}}/2 = 18188.35 \text{ Hz (149.70 ppm)}$$

$$\nu_{\text{B}} = \nu_{\text{M}} + \delta_{\text{AB}}/2 = 18771.55 \text{ Hz (154.50 ppm)}$$

### ABX system

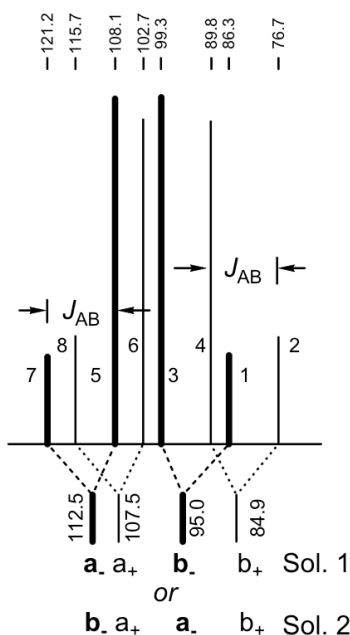
#### Chemical shift of the AB systems

**121.2 ; 115.7 ; 108.1 ; 102.7 ; 99.3 ; 89.8 ; 86.3 ; 76.7**

An ABX system will consist of two superimposed AB sub-spectra, with the same  $J_{\text{AB}}$

**All you need to do is to find the two AB sub-spectra + and -**

- Find the value of  $^2J_{\text{AB}} = \mathbf{13.1 \text{ Hz}}$
- Solve the AB system for each quartet.



$$c_{-} = (5+3)/2 = 103.7$$

$$\Delta\nu_{\text{ab-}} = \delta_{-} = \sqrt{(7-1)(5-3)} = 17.5$$

$$c_{-} \pm \delta_{-}/2 = 103.7 \pm 8.76 = 112.5, 94.9$$

$$c_{+} = (6+4)/2 = 96.25$$

$$\Delta\nu_{\text{ab+}} = \delta_{+} = \sqrt{(8-2)(6-4)} = 22.43$$

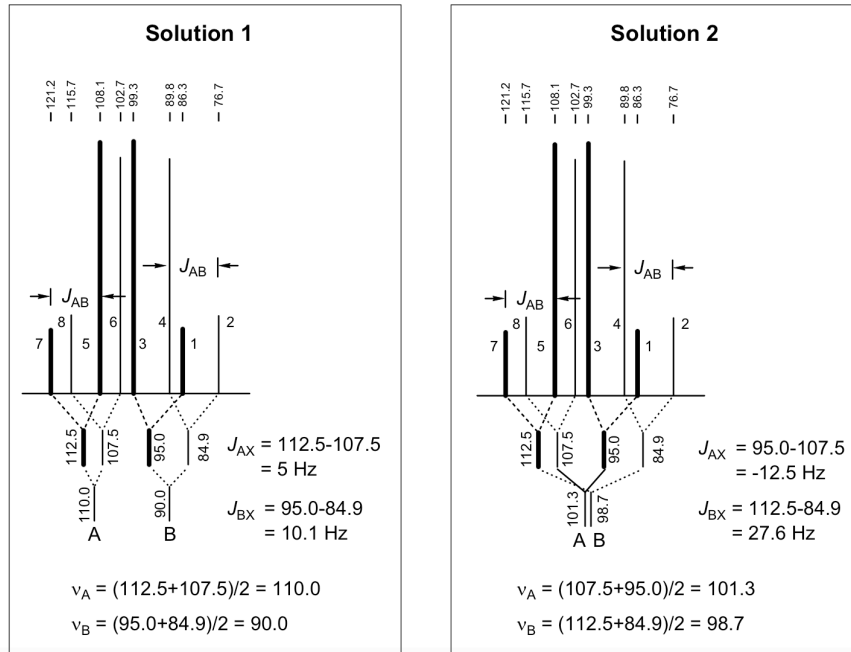
$$c_{+} \pm \delta_{+}/2 = 96.25 \pm 11.21 = 107.5, 85.0$$

At this stage, we know one of the bold lines is **a**, and the other **b**, but we do not know which is which.

Similarly for the thin lines.

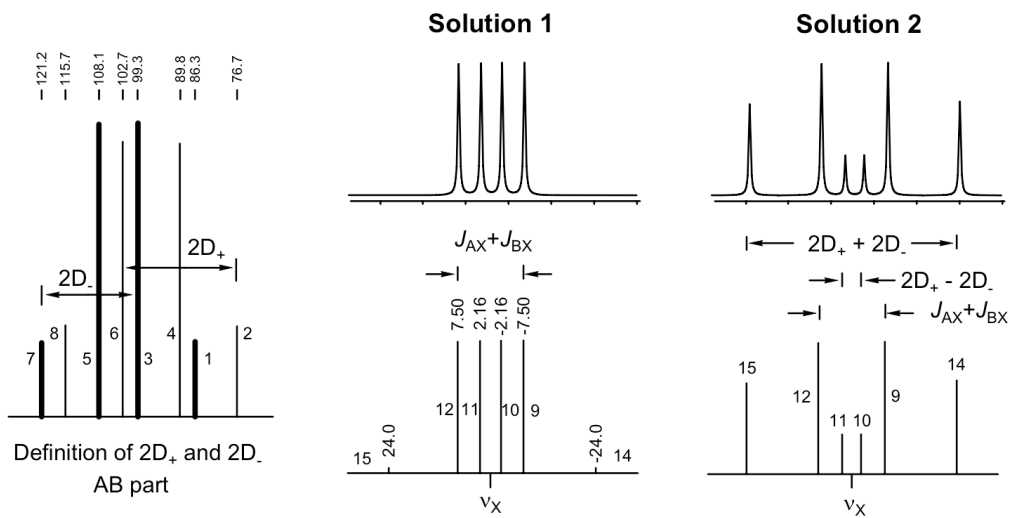
Problem Set - NMR IV - corrections

3. Compare the two solution you have found.



4. Cite some arguments to choose between both. See Lecture slide 8-13
5. The two lines having small intensity are combination lines and decrease in intensity as  $\nu_A - \nu_B$  increases. Their intensity is  $\sin^2(\phi + \phi_-)$  for an intensity of 1 for the external lines of the X system.

a.  $\phi_+$  and  $\phi_-$  can be calculated from the formula  $\sin^2 \phi = J/2C$  (see slide #54) In the solution below  $D = C$



b. Give the calculated intensity of the two combinations lines considering the two solutions obtained before and conclude.

## Problem Set - NMR IV - corrections

### Solution 1

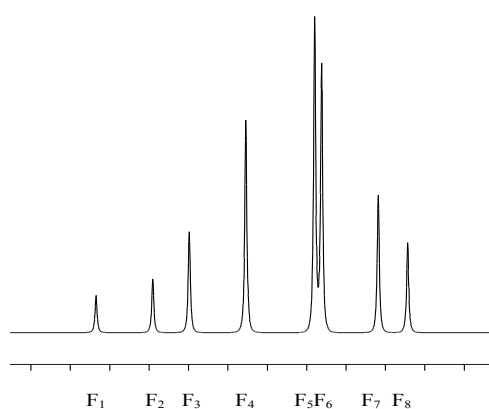
$$\begin{aligned}\Phi_{1+} &= 0.5\arcsin(J_{AB}/2D_+) \\ &= 0.5\arcsin(13.0/26.0) = 15.0 \\ \Phi_{1-} &= 0.5\arcsin(J_{AB}/2D_-) \\ &= 0.5\arcsin(13.0/21.9) = 18.2 \\ i_{10} = i_{11} &= \cos^2(\Phi_{1+} - \Phi_{1-}) \\ &= \cos^2(15.0 - 18.2) = 0.997 \\ i_{14} = i_{15} &= \sin^2(\Phi_{1+} - \Phi_{1-}) \\ &= \sin^2(15.0 - 18.2) = 0.003\end{aligned}$$

$$\arcsin = \sin^{-1}$$

### Solution 2

$$\begin{aligned}\Phi_{2+} &= \Phi_{1+} \\ &= 15.0 \\ \Phi_{2-} &= 90 - \Phi_{1-} \\ &= 90 - 18.2 = 71.8 \\ i_{10} = i_{11} &= \cos^2(\Phi_{2+} - \Phi_{2-}) \\ &= \cos^2(15.0 - 71.8) = 0.30 \\ i_{14} = i_{15} &= \sin^2(\Phi_{2+} - \Phi_{2-}) \\ &= \sin^2(15.0 - 71.8) = 0.70\end{aligned}$$

## AB<sub>2</sub> system



<i>Frequency</i>	<i>Intensity</i>
218.614	0.368
146.692	0.531
100.000	1.000
28.078	2.106
-59.464	3.101
-68.614	2.632
-140.536	1.363
-178.078	0.894

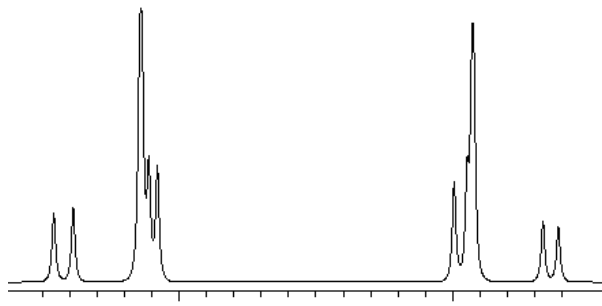
- $\nu_A = 100.00$  Hz
- $\nu_B = 100$  Hz and  $J_{AB} = 100$  Hz

## AA'XX' system

In strict AA'XX' systems spectra are given by pairs of different species:

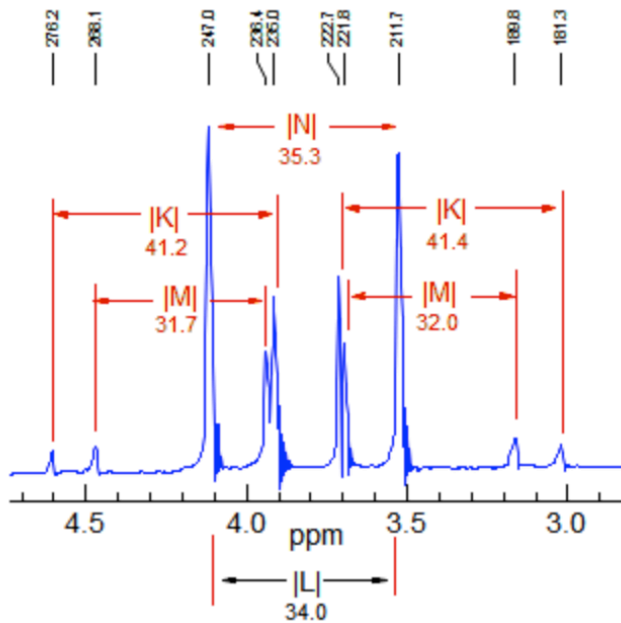
For these systems we define

$ K  =  J_{AA'} + J_{XX'} $	“ <i>J</i> ” of one ab quartet
$ L  =  J_{AX} - J_{AX'} $	“ $\delta$ ” of both ab quartet
$ M  =  J_{AA'} - J_{XX'} $	“ <i>J</i> ” of other ab quartet
$ N  =  J_{AX} + J_{AX'} $	“ <i>J</i> ” of doublet



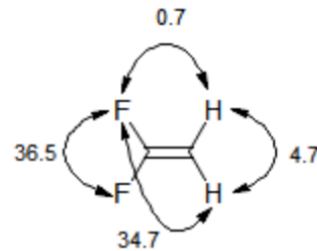
For the molecule 7, the <sup>19</sup>F spectrum shows 10 lines with the following chemical shifts in Hz at:  
276.2 ; 268.1 ; 247.0 ; 236.4 ; 235.0 ; 222.7 ; 221.8 ; 211.7 ; 189.8 ; 181.3.

Problem Set - NMR IV - corrections

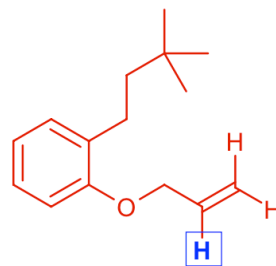
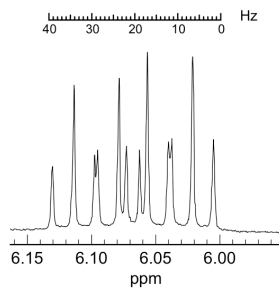


$|K| = |J_{AA'} + J_{XX'}|$  "J" of one ab quartet  
 $|L| = |J_{AX} - J_{AX'}|$  "δ" of both ab quartets  
 $|M| = |J_{AA'} - J_{XX'}|$  "J" of other ab quartet  
 $|N| = |J_{AX} + J_{AX'}|$  "doublet"

If we make the reasonable assumption that  $J_{FF} > J_{HH}$  and  $J_{HF}(\text{trans}) > J_{HF}(\text{cis})$  we get the following values:

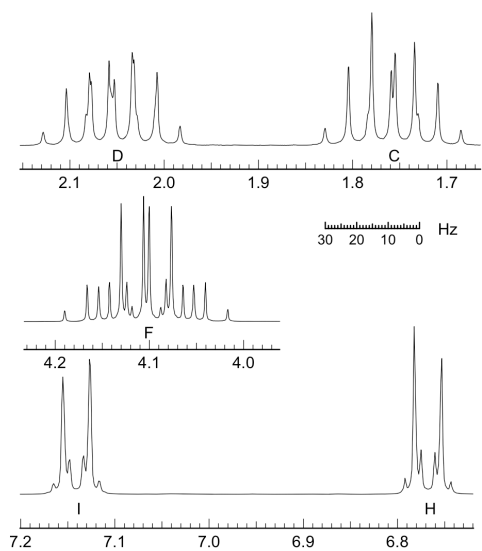


- Determine  $N = 35.3$  Hz
- Measure  $K$  (41.2 and 41.4 Hz) and  $M$  (31.7 Hz and 32.0 Hz).
- Calculate  $L$  from the formulae given in equation 7 For  $K$   $L = 33.8$  Hz and for  $L = 34.2$  Hz.
- Calculate  $J_{AA'}$  and  $J_{XX'}$  by summing and subtracting  $K$  and  $M$ .  
 $J_{AA'} = (K+M)/2 = 36.5$  Hz;  $J_{XX'} = (K-M)/2 = 4.7$  Hz
- Calculate  $J_{AX}$  and  $J_{AX'}$  by summing and subtracting  $L$  and  $N$ .  
 $J_{AX} = (N+L)/2 = 34.7$  Hz;  $J_{AX'} = (N-L)/2 = 0.7$  Hz



ddt, 17, 11, 5

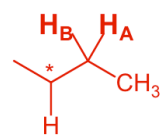
# Problem Set - NMR IV - corrections



ABM<sub>3</sub>Y

$\delta$  2.05, d quintets,  $J = 13.4, 7.5$

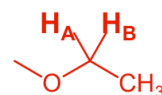
$\delta$  1.75, d quintets,  $J = 13.5, 7.5$



ABX<sub>3</sub>

$\delta$  4.13, dq,  $J = 11, 7$

$\delta$  4.08, dq,  $J = 11, 7$



$R_1, R_2$  must be electron donating

AA'XX' (or AA'BB')

